

Strip telescope

Some tries to estimate the telescope resolution

(Auguste Besson & Jérôme Baudot)

$$\sigma_{\text{Residual}}^2 = \sigma_{\text{telescope}}^2 + \sigma_{\text{DUT}}^2 + \sigma_{\text{M.S.}}^2$$

Method 1 : chi-2 law

chi-2 law

- Goal: obtain the strip resolution
- Selection of reconstructed tracks

- the χ^2 should obey to a χ^2 law:
- In principal:

$$f(t) = \frac{t^{k/2-1} \times e^{-\alpha t/2}}{2^{k/2} \times \Gamma(k/2)}$$

- ✓ k = Number of Degrees of Freedom:
- ✓ Γ = gamma function
- ✓ $\alpha = 1$ if the uncertainty is correct

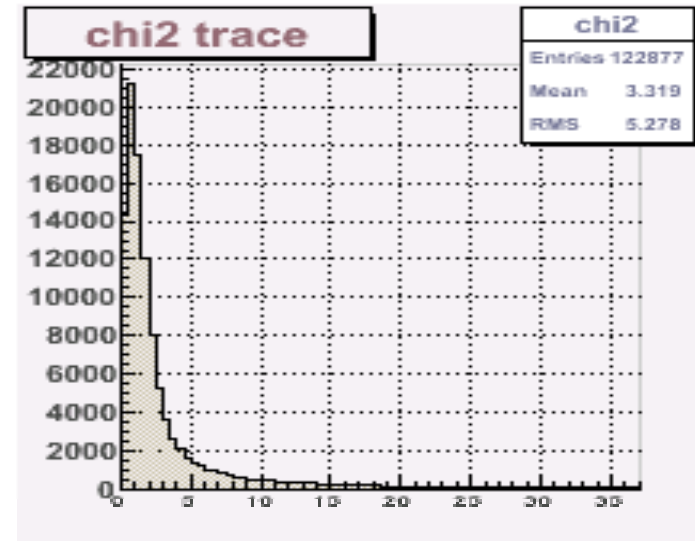
- k = (4 points – 2 parametes of the straight line) x 2 dimensions = 4 ?

- Fit the function with α, k, norm

- True resolution:

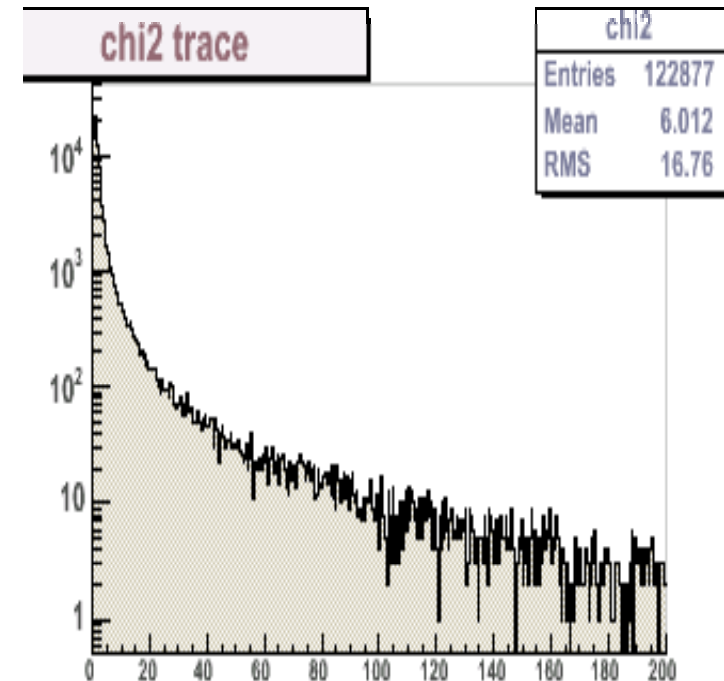
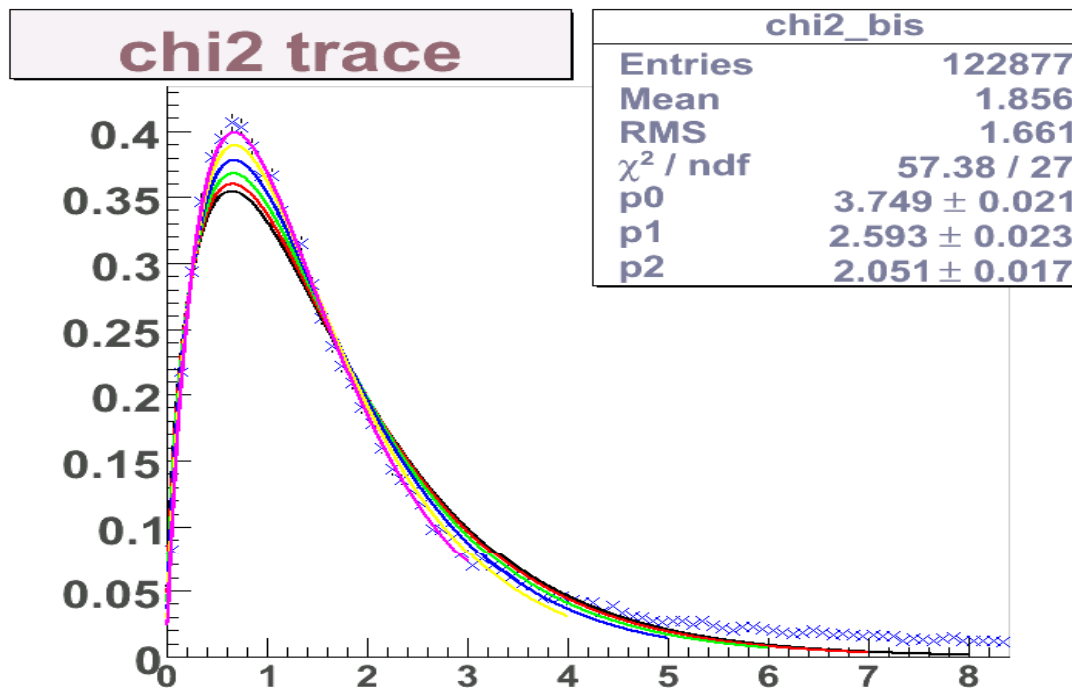
$$\sigma_{\text{true}} = \sigma_{\text{used}} / \sqrt{\alpha}$$

- 1^{ere} step: Reconstruct data with an assumed resolution of 2 μm



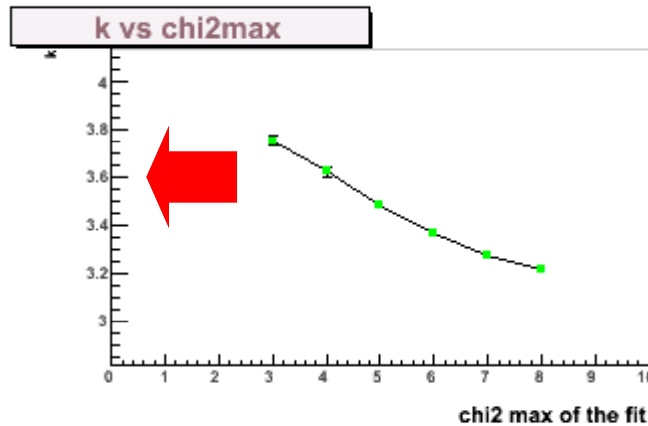
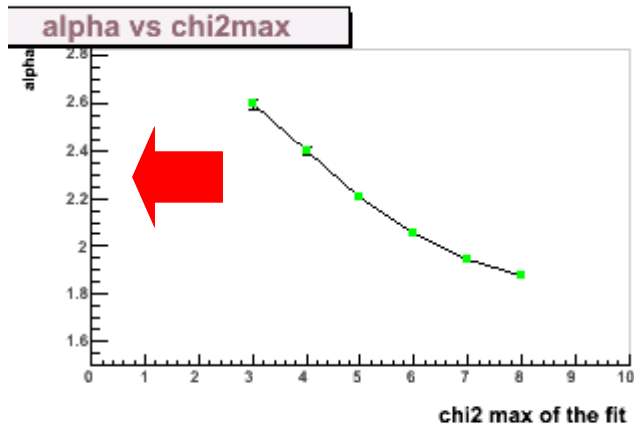
Result of the fit

- Problem : the fit depends on the maximum range
 - The data doesn't follow exactly a χ^2 law
 - ✓ multiple scattering ?
 - ✓ No homogeneous resolution in each plane ?



Results for $\sigma_{\text{used}} = 2 \mu\text{m}$

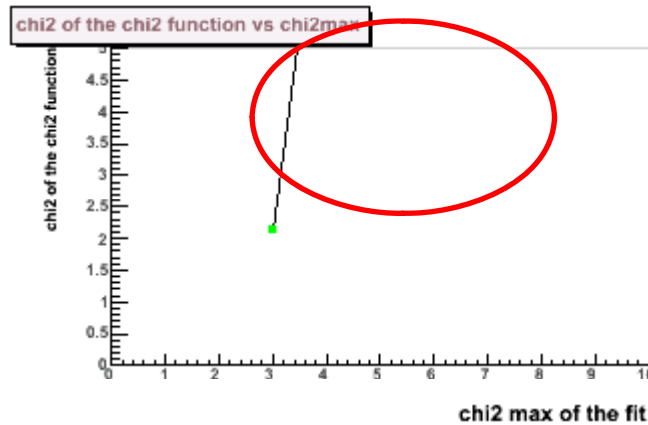
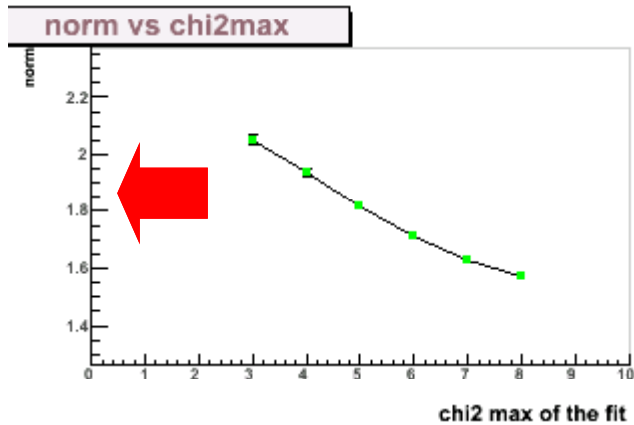
- α , k , norm : non constants depending on the maximum range of the fit.



$$\alpha \sim 2$$

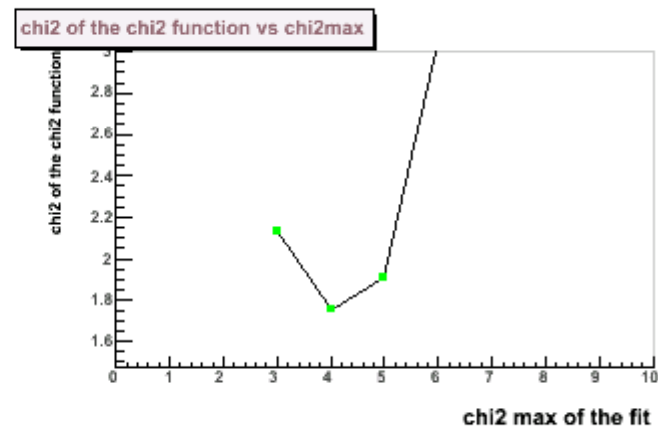
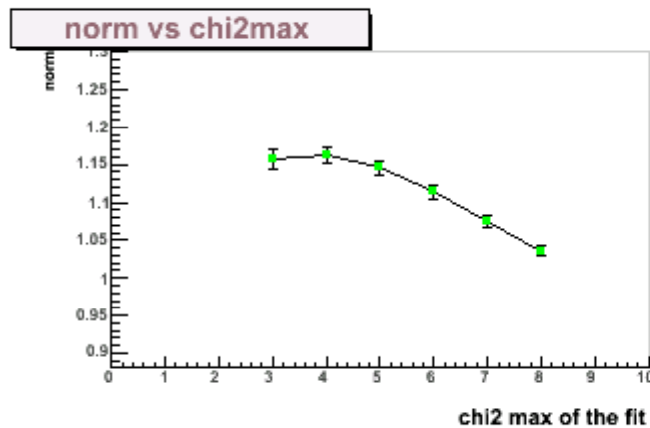
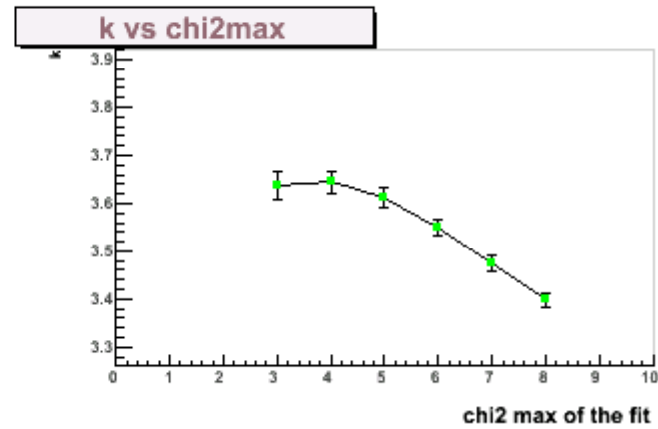
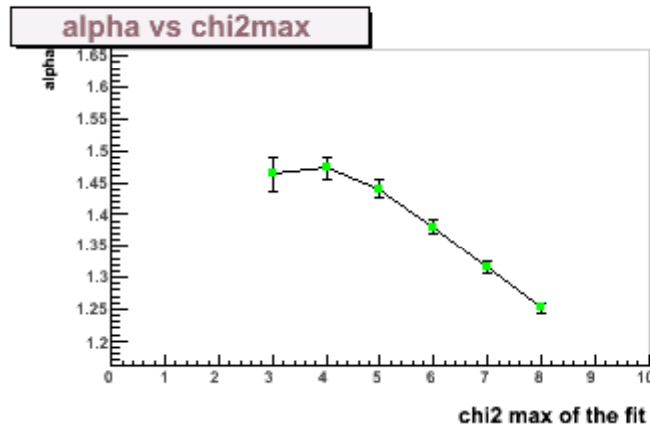
$$\sigma_{\text{true}} \sim \sigma_{\text{used}} / \sqrt{\alpha}$$

$$\sigma_{\text{true}} \sim 1.4$$

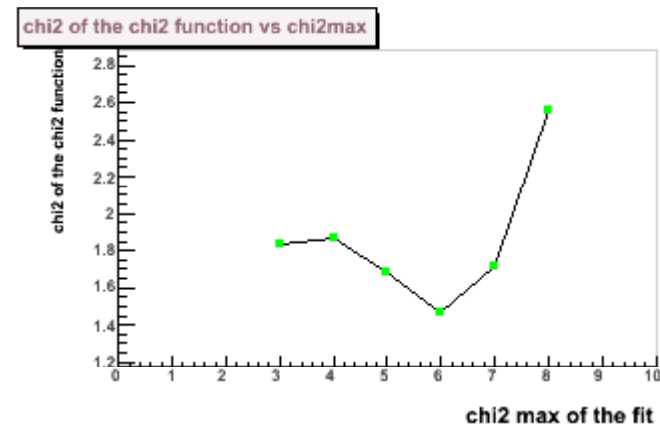
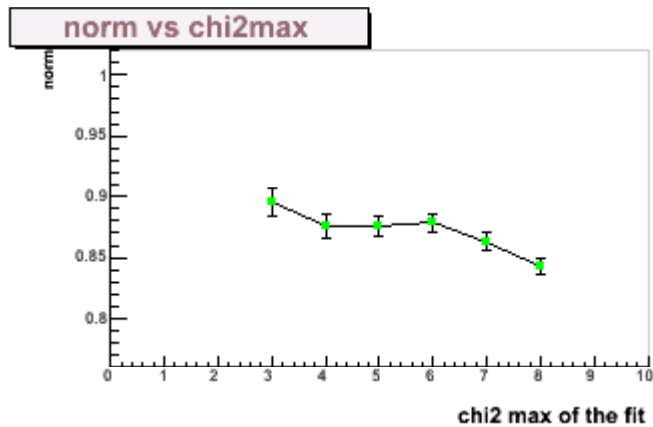
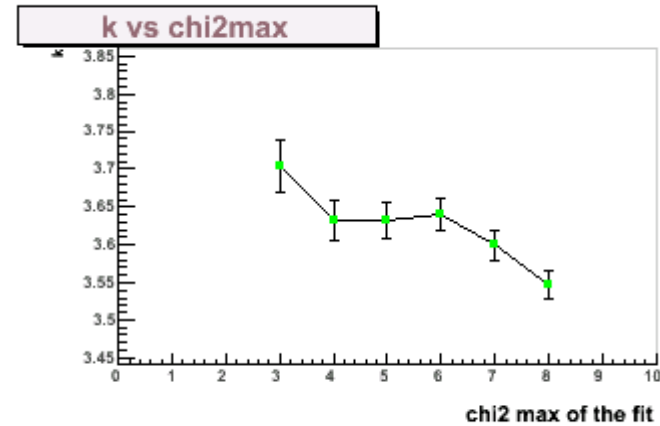
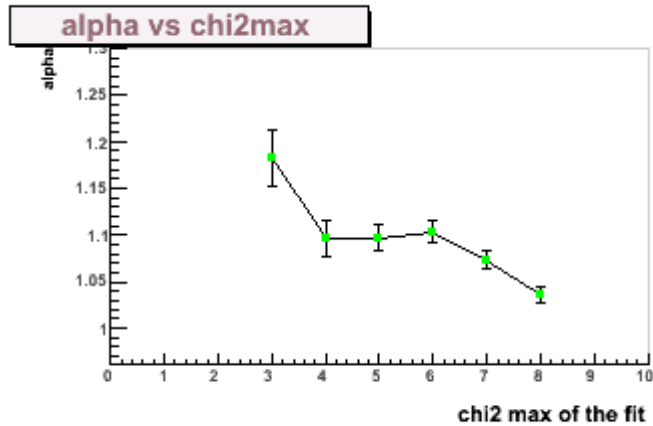


Start again with
 $\sigma_{\text{used}} \sim 1.5$

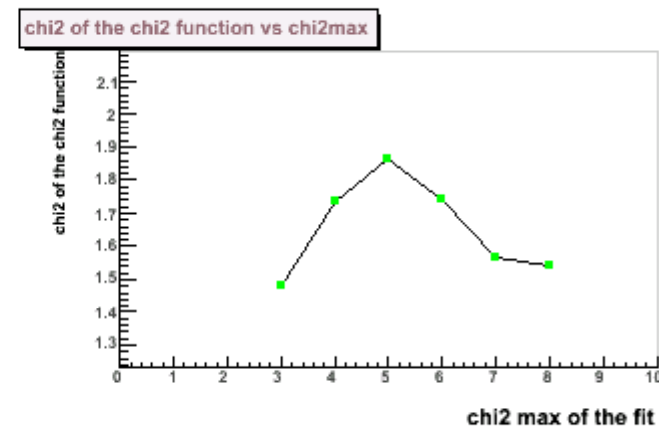
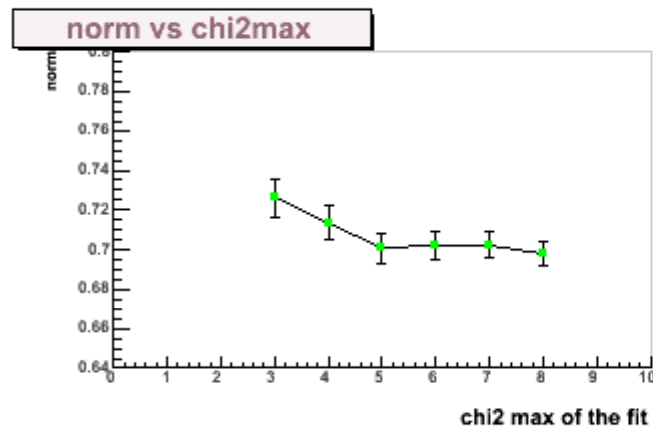
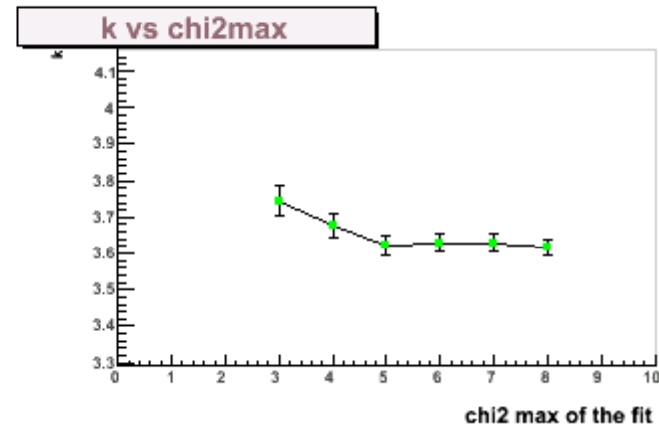
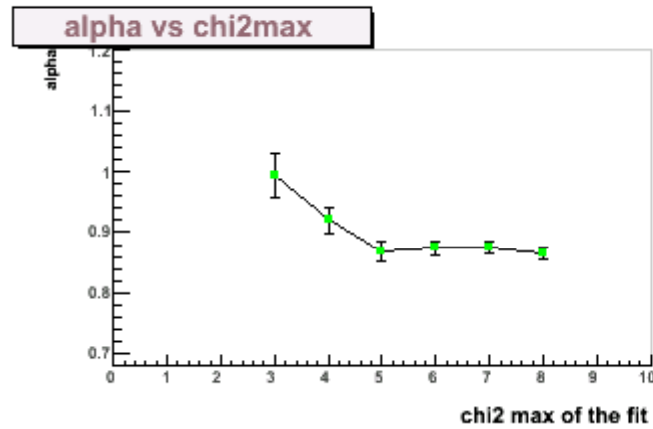
Results for $\sigma_{\text{used}} = 1.5 \mu\text{m}$



Results for $\sigma_{\text{used}} = 1.3 \mu\text{m}$



Results for $\sigma_{\text{used}} = 1.16 \mu\text{m}$



Summary: σ which fits best $\sim 1.2\text{-}1.3 \mu\text{m}$

Assumed resolution = 1.16 μm				
Max range of the fit	α	k	norm	χ^2/NdF of the fit
3	0.99 \pm 0.008	3.7 \pm 0.02	0.72 \pm 0.010	1.54
4	0.92 \pm 0.008	3.7 \pm 0.02	0.71 \pm 0.009	1.54
5	0.87 \pm 0.008	3.6 \pm 0.02	0.70 \pm 0.008	1.54
6	0.87 \pm 0.015	3.6 \pm 0.02	0.70 \pm 0.008	1.54
7	0.87 \pm 0.010	3.6 \pm 0.02	0.70 \pm 0.006	1.56
8	0.87 \pm 0.008	3.6 \pm 0.02	0.70 \pm 0.006	1.54
Assumed resolution = 1.5 μm				
Max range of the fit	α	k	norm	χ^2/NdF of the fit
3	1.46 \pm 0.026	3.6 \pm 0.03	1.16 \pm 0.013	2.13
4	1.47 \pm 0.017	3.6 \pm 0.02	1.16 \pm 0.011	1.75
5	1.44 \pm 0.013	3.6 \pm 0.02	1.15 \pm 0.010	1.91
6	1.38 \pm 0.011	3.5 \pm 0.02	1.11 \pm 0.009	3.05
7	1.32 \pm 0.010	3.5 \pm 0.02	1.07 \pm 0.008	5.09
8	1.25 \pm 0.008	3.4 \pm 0.02	1.03 \pm 0.007	8.43

Assumed resolution = 1.3 μm				
Max range of the fit	α	k	norm	χ^2/NdF of the fit
3	1.18 \pm 0.030	3.7 \pm 0.03	0.90 \pm 0.011	1.83
4	1.09 \pm 0.019	3.6 \pm 0.03	0.88 \pm 0.010	1.87
5	1.09 \pm 0.014	3.6 \pm 0.02	0.88 \pm 0.008	1.69
6	1.10 \pm 0.011	3.6 \pm 0.02	0.87 \pm 0.008	1.46
7	1.07 \pm 0.010	3.6 \pm 0.02	0.86 \pm 0.007	1.71
8	1.04 \pm 0.008	3.5 \pm 0.02	0.84 \pm 0.007	2.55
Assumed resolution = 2.0 μm				
Max range of the fit	α	k	norm	χ^2/NdF of the fit
3	2.59 \pm 0.023	3.7 \pm 0.02	2.05 \pm 0.017	2.12
4	2.40 \pm 0.017	3.6 \pm 0.02	1.93 \pm 0.014	8.3
5	2.20 \pm 0.015	3.5 \pm 0.02	1.81 \pm 0.013	18
6	2.05 \pm 0.013	3.4 \pm 0.02	1.71 \pm 0.012	31
7	1.94 \pm 0.013	3.3 \pm 0.01	1.63 \pm 0.012	42
8	1.88 \pm 0.013	3.2 \pm 0.01	1.57 \pm 0.011	50

Method 2 : from the digital resolution

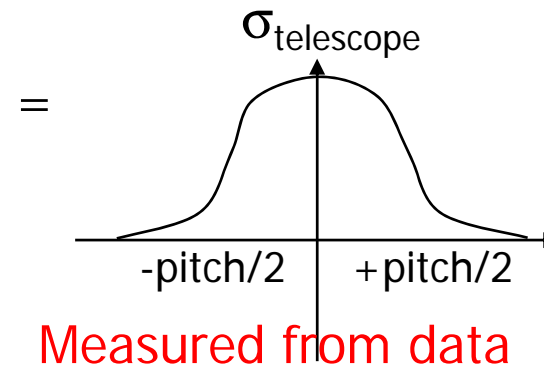
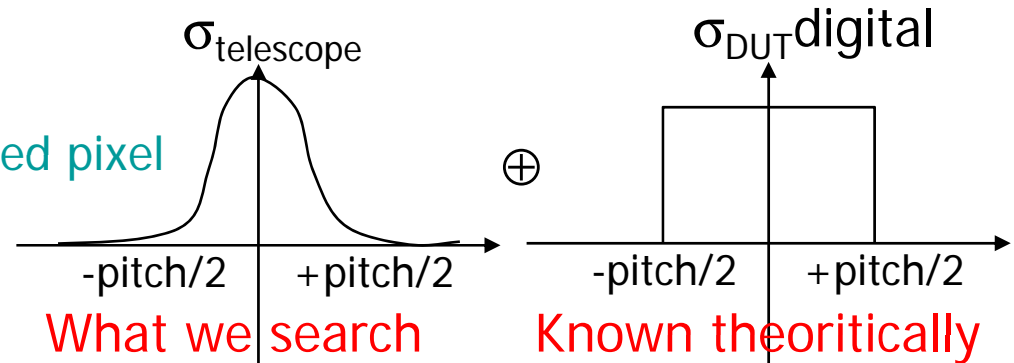
Residual with the digital position

- $\sigma^2_{\text{residual}} = \sigma^2_{\text{telescope}} + \sigma^2_{\text{DUT}}$
 - Digital Position = center of the seed pixel
- Need to deconvolute ($p = \text{pitch}$)

$$f(r) = \int_{r-p/2}^{r+p/2} f_{\text{DUT}}(r-\varepsilon) \times f_{\text{telescope}}(\varepsilon) d\varepsilon$$

↑
1/p

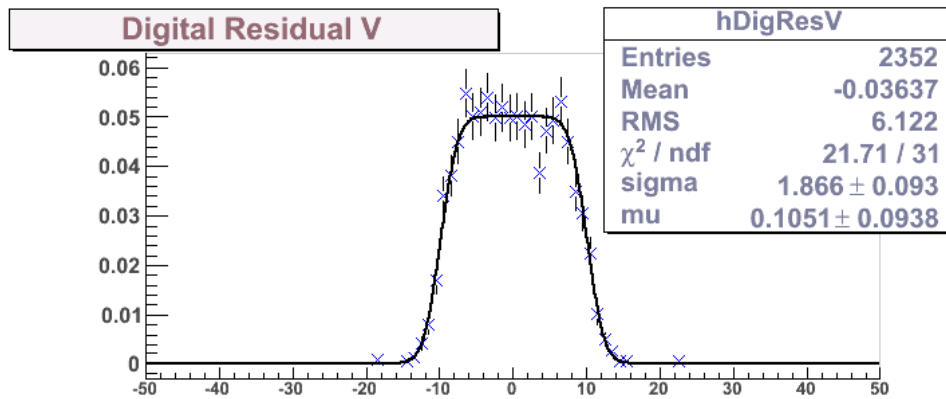
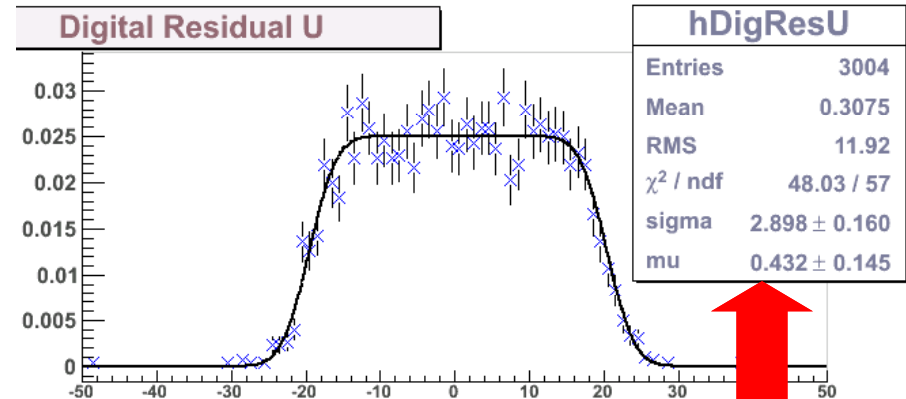
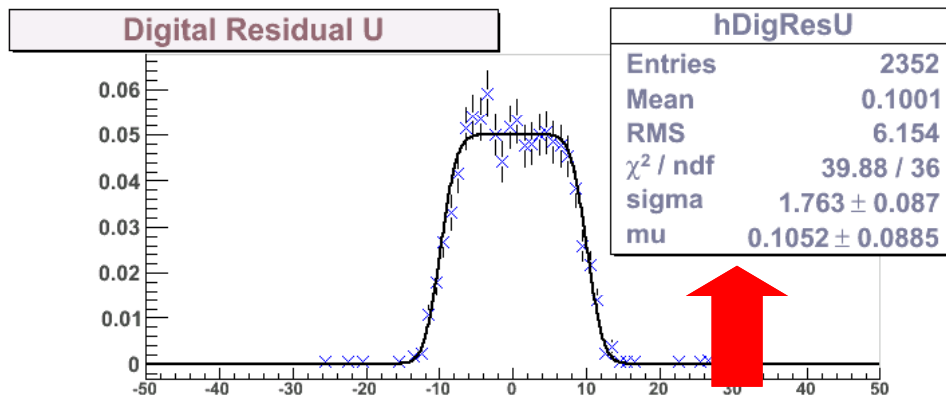
↑
 $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\varepsilon-\mu)^2}{2\sigma^2}}$



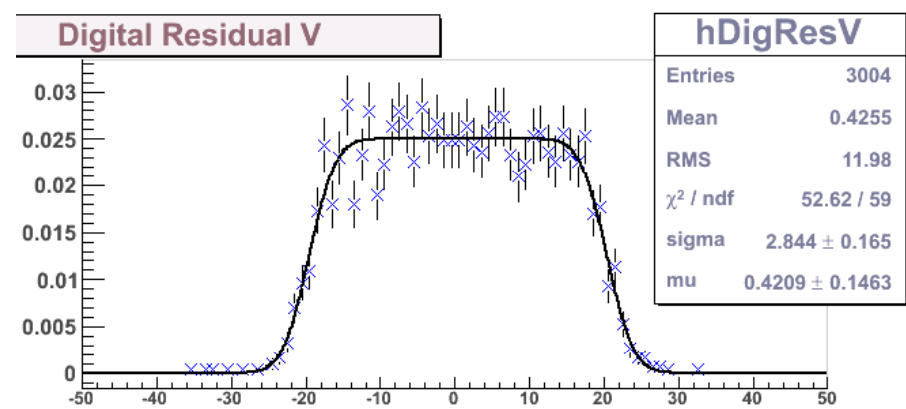
- Fit from the data
 - Parameters : σ, μ

Results for M9, pitch 20 μm & 40 μm

- Good chi-2
- Problem: σ is not constant !! (1.8 ou 2.8 μm ???)
 - There is another source of uncertainty which degrades the residual (seed choice ?)



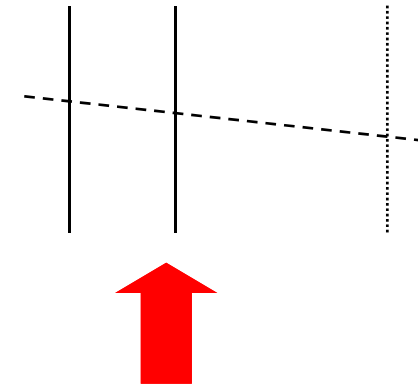
Pitch = 20 μm



Pitch = 40 μm

Other methods

- Dirk Meier thesis: (RD42)
 - Estimate from the residual = $1.35 \mu\text{m} = 1.93 / \sqrt{2}$ (no uncertainty given)
 - BUT: residual fitted in a $\pm 4 \mu\text{m}$ range \Rightarrow residual = $1.93 \mu\text{m}$
- Estimate the resolution from the Alignment residual
 - Residual with « long tails »
 - Residual fitted in a range of $\pm 10 \mu\text{m}$
 - ✓ σ_{residual} in one plane $\sim 2.5\text{-}3.7 \mu\text{m} \sim \sigma_{\text{spatial}}/\sqrt{2}$
 - ✓ $\sigma_{\text{spatial}} \sim 1.7\text{-}2.6 \mu\text{m}$
 - Residual fitted in a range of $\pm 4 \mu\text{m}$
 - ✓ σ_{residual} in one plane $\sim 2.2\text{-}3.5 \mu\text{m} \sim \sigma_{\text{spatial}}/\sqrt{2}$
 - ✓ $\sigma_{\text{spatial}} \sim 1.5\text{-}2.5 \mu\text{m}$
 - Spatial resolution = $2 \pm 0.5 \mu\text{m}$
(classical method)



Summary

- chi-2 method:
 - ~ 1.2-1.3 $\mu\text{m} \pm \text{xxx}$
 - why k ~3.5 and not 4 ?
 - Multiple scattering ?
 - Resolution not constant between planes and inside a plane ?
- Digital position method
 - ~ between 1.8 and 3 $\mu\text{m} \dots$
- Other estimates:
 - D.Meier's thesis: 1.35 $\mu\text{m} \pm \text{xxx}$
 - Residual of the alignment: ~ 2 \pm 0.5 μm

 - There are additional effects which make these methods incompatible (alignment, moves, inhomogeneity, different plans, etc.)
 - A better estimate needs a dedicated study
 - Our uncertainty on the DUT resolution ~ few $10^{-1} \mu\text{m}$
 - Which uncertainty do we want on the DUT resolution ?