Strip telescope

Some tries to estimate the telescope resolution

(Auguste Besson & Jérôme Baudot)

\[ \sigma_{\text{Residual}}^2 = \sigma_{\text{telescope}}^2 + \sigma_{\text{DUT}}^2 + \sigma_{\text{M.S.}}^2 \]
Method 1: chi-2 law
Chi-2 law

- Goal: obtain the strip resolution
- Selection of reconstructed tracks
  - The $\chi^2$ should obey to a $\chi^2$ law:
  - In principal:
    \[
    f(t) = \frac{t^{k/2-1} \cdot e^{-\alpha t/2}}{2^{k/2} \cdot \Gamma(k/2)}
    \]
    - $k =$ Number of Degrees of Freedom:
    - $\Gamma =$ gamma function
    - $\alpha =$ 1 if the uncertainty is correct
  - $k =$ (4 points – 2 parameters of the straight line ) x 2 dimensions = 4 ?
  - Fit the function with $\alpha, k, \text{norm}$
  - True resolution: $\sigma_{\text{true}} = \sigma_{\text{used}} / \sqrt{\alpha}$
  - 1ère step: Reconstruct data with an assumed resolution of 2 µm
Result of the fit

• Problem: the fit depends on the maximum range
  ➢ The data doesn’t follow exactly a $\chi^2$ law
  ✓ multiple scattering?
  ✓ No homogeneous resolution in each plane?
Results for $\sigma_{\text{used}} = 2 \mu m$

- $\alpha$, $k$, norm: non constants depending on the maximum range of the fit.

$\alpha \sim 2$

$\sigma_{\text{true}} \sim \sigma_{\text{used}} / \sqrt{\alpha}$

$\sigma_{\text{true}} \sim 1.4$

Start again with $\sigma_{\text{used}} \sim 1.5$
Results for $\sigma_{\text{used}} = 1.5 \, \mu m$
Results for $\sigma_{\text{used}} = 1.3 \, \mu m$
Results for $\sigma_{used} = 1.16 \, \mu m$
Summary: σ which fits best ∼1.2-1.3 µm

<table>
<thead>
<tr>
<th>Max range of the fit</th>
<th>Assumed resolution = 1.16µm</th>
<th>Assumed resolution = 1.3µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>k</td>
<td>norm</td>
</tr>
<tr>
<td>3</td>
<td>0.99±0.008</td>
<td>3.7±0.02</td>
</tr>
<tr>
<td>4</td>
<td>0.92±0.008</td>
<td>3.7±0.02</td>
</tr>
<tr>
<td>5</td>
<td>0.87±0.008</td>
<td>3.6±0.02</td>
</tr>
<tr>
<td>6</td>
<td>0.87±0.015</td>
<td>3.6±0.02</td>
</tr>
<tr>
<td>7</td>
<td>0.87±0.010</td>
<td>3.6±0.02</td>
</tr>
<tr>
<td>8</td>
<td>0.87±0.008</td>
<td>3.6±0.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Max range of the fit</th>
<th>Assumed resolution = 1.5µm</th>
<th>Assumed resolution = 2.0µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>k</td>
<td>norm</td>
</tr>
<tr>
<td>3</td>
<td>1.46±0.026</td>
<td>3.6±0.03</td>
</tr>
<tr>
<td>4</td>
<td>1.47±0.017</td>
<td>3.6±0.02</td>
</tr>
<tr>
<td>5</td>
<td>1.44±0.013</td>
<td>3.6±0.02</td>
</tr>
<tr>
<td>6</td>
<td>1.38±0.011</td>
<td>3.5±0.02</td>
</tr>
<tr>
<td>7</td>
<td>1.32±0.010</td>
<td>3.5±0.02</td>
</tr>
<tr>
<td>8</td>
<td>1.25±0.008</td>
<td>3.4±0.02</td>
</tr>
</tbody>
</table>
Method 2: from the digital resolution
Residual with the digital position

- \( \sigma_{\text{residual}}^2 = \sigma_{\text{telescope}}^2 + \sigma_{\text{DUT}}^2 \)

  - Digital Position = center of the seed pixel

- Need to deconvolute (\( p = \text{pitch} \))

\[
f(r) = \int_{r-p/2}^{r+p/2} f_{\text{DUT}}(r-\varepsilon) \times f_{\text{telescope}}(\varepsilon) d\varepsilon
\]

\[
\frac{1}{p} \times \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(\varepsilon-\mu)^2}{2\sigma^2}}
\]

- Fit from the data
  - Parameters: \( \sigma, \mu \)
Results for M9, pitch 20 µm & 40 µm

- Good chi-2
- Problem: $\sigma$ is not constant !! (1.8 ou 2.8 µm ???)
  - There is another source of uncertainty which degrades the residual (seed choice ?)
Other methods

- Dirk Meier thesis: (RD42)
  - Estimate from the residual = 1.35 µm = 1.93 / √2 (no uncertainty given)
  - BUT: residual fitted in a ±4 µm range ⇒ residual = 1.93 µm

- Estimate the resolution from the Alignment residual
  - Residual with « long tails »
  - Residual fitted in a range of ±10 µm
    - σ_{residual} in one plane ~ 2.5-3.7 µm ~ σ_{spatial}/√2
    - σ_{spatial} ~1.7-2.6 µm
  - Residual fitted in a range of ±4 µm
    - σ_{residual} in one plane ~ 2.2-3.5 µm ~ σ_{spatial}/√2
    - σ_{spatial} ~1.5-2.5 µm
  - Spatial resolution = 2 ± 0.5 µm
    (classical method)
Summary

• chi-2 method:
  ➢ $\sim 1.2\text{-}1.3 \, \mu m \pm xxx$
  ➢ why $k \sim 3.5$ and not 4 ?
  ➢ Multiple scattering ?
  ➢ Resolution not constant between planes and inside a plane ?

• Digital position method
  ➢ $\sim$ between 1.8 and 3 $\mu m$ …

• Other estimates:
  ➢ D.Meier’s thesis: 1.35 $\mu m \pm xxx$
  ➢ Residual of the alignment: $\sim 2 \pm 0.5 \, \mu m$

➢ There are additionnal effects which make these methods incompatible
  (alignment, moves, inhomogeneity, different plans, etc.)
  ➢ A better estimate needs a dedicated study
  ➢ Our uncertainty on the DUT resolution $\sim$ few $10^{-1} \, \mu m$
  ➢ Which uncertainty do we want on the DUT resolution ?