Quantum Field Theory

2019

Problem 1

Momentum of scalar particles (8 points):

One considers neutral non-interactive scalar particles described by the Lagrangian

$$L = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2}\phi^2.$$

The field operator is of the form

$$\phi(\mathbf{x}, t) = \int \frac{d^3k}{(2\pi)^3 2\omega_k}\left[a(\mathbf{k})e^{-i\omega_\mathbf{k}t + i\mathbf{k}\cdot\mathbf{x}} + a^\dagger(\mathbf{k})e^{i\omega_\mathbf{k}t - i\mathbf{k}\cdot\mathbf{x}}\right],$$

where $\omega_\mathbf{k} = \sqrt{m^2 + \mathbf{k}^2}$ with

$$[a(\mathbf{k}), a(\mathbf{k}')] = [a^\dagger(\mathbf{k}), a^\dagger(\mathbf{k}')] = 0, \quad \left[a(\mathbf{k}), a^\dagger(\mathbf{k}')\right] = (2\pi)^3 2\omega_k \delta(\mathbf{k} - \mathbf{k}').$$

a: Give the expression of the momentum,

$$P^j = \int d^3x T^{0j}(t, \mathbf{x}), \tag{1}$$

defined by the help of the energy-momentum tensor

$$T^{\mu\nu} = \frac{\partial L}{\partial \partial_\mu \phi}\partial^\nu \phi - g^{\mu\nu} L,$$

in terms of the operators $a(\mathbf{k})$ and $a^\dagger(\mathbf{k})$.

b: Show that the state $(a^\dagger(\mathbf{p}))^2|0\rangle$ is an eigenstate of the momentum operator (1) and find its eigenvalue.

Problem 2

Quantum electrodynamics (8 points):

The Lagrangian of QED is

$$L = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{\lambda}{2}(\partial^\mu A_\mu)^2 + \bar{\psi}[i\gamma^\mu(\partial_\mu - ieA_\mu) - m]\psi.$$

a: What are the free and the interaction Lagrangians?

b: Draw the $\mathcal{O}(e^2)$ Feynman graph contributing to the electron propagator $\langle 0|T[\psi_a(x)\bar{\psi}_b(y)]|0\rangle$ and give its mathematical expression in terms of the photon and the electron propagators as a space-time integral over the vertices. Indicate explicitly the Dirac and the vector indices.

c: Draw the connected $\mathcal{O}(e^6)$ Feynman graphs of the scattering amplitude $\gamma + \gamma \to e^- + e^+$.

Problem 3

General questions (4 points):

a: Why do we use harmonic oscillators in quantum field theory to represent the particles?

b: Why do we use the interaction (Dirac) representation in quantum field theory?