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# GENERALIZATION OF THE MASSIVE SCALAR MULTIPLET COUPLING TO THE SUPERGRAVITY

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ABSTRACT

It is shown that the local supersymmetric coupling of the scalar matter multiplet to the supergravity depends on an arbitrary function. This function can be chosen such that the value of the cosmologic term is zero at the energy minimum.

АННОТАЦИЯ

Локальная суперсимметрическая связь скалярного материального мультиплета с супергравитацией зависит от произвольной функции. Эту функцию можно выбрать таким образом, чтобы космологический член исчезал при минимуме энергии.

KIVONAT

A skalár anyag multiplettnek a szupergravitációhoz való lokális szuperszimmetrikus csatolása tartalmaz egy tetszőleges függvényt. Ezt a függvényt meg lehet ugy választani, hogy az energiaminimumban a kozmológiai tag nulla legyen.

Supergravity [1-3] is a locally supersymmetric theory of the gravitation. The model building strategy starts from a heuristic field theoretical realisation of certain graded Lie - algebra [1], which somewhat related to the conventional minimal coupling Lagrangian including this fields. Then the local supersymmetry of the Lagrangian can be achieved by adding successively terms to the Lagrangian and to the expression of the supersymmetric transformations [3,4].

In this manner Das et al. [5] have given a Lagrangian of the supersymmetricaly coupled scalar massive matter to the gravity supermultiplet. It contains spin 2 vierbein field $e_\mu^a$ ,spin 3/2 gauge field $\tau_\mu$ , spin 1/2 matter field $\chi$ , and scalar and pseudoscalar matter field A and B, respectively. All of these fields are real.

In this paper [6] it is shown, that the class of the supersymmetric Lagrangian is larger than the one described in [5]. Now we introduce a more general Lagrangian and that we present some physically interesting models, as special cases.

Let us denote the Lagragian by $\mathcal{L} = \mathcal{L}_o + \mathcal{L}_m + \mathcal{L}_i$, [7]

$$\mathcal{L}_o = -(2K^2)^{-1} eR(e) - \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\tau}_\mu\gamma_5\gamma_\nu D_\rho\tau_\sigma$$

$$- \frac{1}{32}eK^2[(\bar{\tau}^b\gamma^a\tau^c)(\bar{\tau}_b\gamma_a\tau_c) + 2(\bar{\tau}^b\gamma^a\tau^c)(\bar{\tau}_a\gamma_b\tau_c)]$$

$$+ \frac{1}{8}eK^2(\bar{\tau}_a\gamma\cdot\tau)^2, \qquad /1/$$

$$\mathcal{L}_m = -\frac{1}{2}eg^{\mu\nu}(\partial_\mu A\partial_\nu A+\partial_\mu B\partial_\nu B) - \frac{1}{2}e\bar{\chi}\gamma^\mu D_\mu\chi$$

$$+ \frac{1}{2}eK\bar{\tau}_\mu(\partial\!\!\!/ A+i\gamma_5\partial\!\!\!/ B)\gamma^\mu\chi$$

$$+ K^2\varepsilon^{\mu\nu\rho\sigma}(\bar{\tau}_\mu\gamma_\nu\tau_\rho)[\frac{1}{32}(\bar{\chi}\gamma_5\gamma_\sigma\chi) + \frac{i}{8}(A\partial_\sigma B)]$$

$$+ eK^2(\bar{\chi}\gamma_5\gamma^\mu\chi)\left[\frac{i}{8}(A\overleftrightarrow{\partial}_\mu B) - \frac{1}{16}(\bar{\tau}_\alpha\gamma_5\gamma_\mu\tau^\alpha) - \frac{1}{64}(\bar{\chi}\gamma_5\gamma_\mu\chi)\right],$$

$$\mathcal{L}_i = -eU - \frac{1}{2}e\bar{\chi}\hat{G}\chi+\frac{1}{2}eK\bar{\tau}_\mu\hat{A}\gamma^\mu\chi+\frac{1}{4}eK\bar{\tau}_\mu\sigma^{\mu\nu}\hat{V}\tau_\nu ,$$

where $e = \det \; e^a_\mu$ $R(e)$ the curvature, $D_\mu$ denotes the gravitational covariant derivative, U and $\hat{A}$, $\hat{G}$, $\hat{V}$ are functions of KA, KB and $i\gamma_5$; $\partial\!\!\!/ \equiv \gamma^\mu \partial_\mu$.

This Lagrangian is invariant under the following supersymmetric transformation:

$$\delta A = \bar{\varepsilon}\chi, \quad \delta B = i\bar{\varepsilon}\gamma_5\chi, \quad \delta e^a_\mu = K\bar{\varepsilon}\gamma^a\tau_\mu ,$$

$$\delta\chi = \delta_1\chi + \delta_2\chi , \quad \delta\tau_\mu = \delta_1\tau_\mu + \delta_2\tau_\mu ,$$

$$\delta_1\chi = \partial\!\!\!/(A+i\gamma_5 B)\varepsilon - \frac{K}{2}\gamma^\mu\varepsilon(\bar{\tau}_\mu\chi) - \frac{K}{2}\gamma_5\gamma^\mu\varepsilon(\bar{\tau}_\mu\gamma_5\chi) + \frac{K^2}{4}\gamma_5\chi\left[\bar{\varepsilon}(A+i\gamma_5 B)\gamma_5\chi\right], \qquad /2/$$

$$\delta_1\tau_\mu = 2K^{-1}D_\mu\varepsilon + \frac{K}{4}\sigma^{ab}\varepsilon\left[2\bar{\tau}_\mu\gamma_a\tau_b + \bar{\tau}_a\gamma_\mu\tau_b\right]$$

$$- \frac{i}{2}K\gamma_5\varepsilon(A\overleftrightarrow{\partial}_\mu B) + \frac{K}{4}\sigma_{\mu\rho}\gamma_5\varepsilon(\bar{\chi}\gamma_5\gamma^\rho\chi) - \frac{K^2}{4}\gamma_5\tau_\mu\left[\bar{\varepsilon}(A+i\gamma_5 B)\gamma_5\chi\right],$$

$$\delta_2\chi = -\hat{A}\varepsilon, \qquad \gamma_2\tau_\mu = \frac{1}{4}\gamma_\mu \hat{V}\varepsilon ,$$

provided that

$$\hat{V} = \alpha e^{\frac{u}{4}},$$

$$\hat{A} = \frac{1}{2}(\alpha' + \frac{1}{2}\bar{Z}\alpha)e^{\frac{u}{4}} ,$$

$$\hat{G} = \frac{1}{2}K(\alpha'' + Z\alpha' + \frac{1}{4}\bar{Z}^2\alpha)e^{\frac{u}{4}}, \qquad /3/$$

$$U = \left[\frac{1}{16}\alpha\bar{\alpha}(\frac{u}{2}-3) + 2\alpha'\bar{\alpha}' + \bar{\alpha}'\bar{Z}\alpha + \alpha'Z\bar{\alpha}\right]e^{\frac{u}{2}},$$

where $\alpha$ is an arbitrary analytic real /in order to preserve the Majorana character of the spinors/ function of $Z = K(A+i\gamma_5 B)$. The prim denotes the derivative with respect to $Z$ , and $\bar{Z} = K(A-i\gamma_5 B)$, $u = Z\bar{Z}$.
/The model described in [5] corresponds to the choice

$$\alpha = Z^2(mK^{-1} + gK^{-2}Z) .$$

The Lagrangian and the transformation laws have been found by the same functional method as in [5] which allows to handle the nonpolynomial Lagrangian of the scalar fields.

The commutators of two supersymmetric transformation as in [4] are given by

$$[\delta_1\delta_2]A = \xi^\mu\delta_\mu A + \delta_{\varepsilon'}A,$$

$$[\delta_1\delta_2]B = \xi^\mu\delta_\mu B + \delta_{\varepsilon'}B$$

$$[\delta_1\delta_2]\chi = \xi^\mu D_\mu\chi + \tfrac{1}{2}\Lambda_{ab}\sigma^{ab}\chi + \delta_{\varepsilon'}\chi, \qquad /4/$$

$$[\delta_1\delta_2]e_{a\mu} = \xi^\nu\partial_\nu e_{a\mu} + (\partial_\mu\xi^\nu)e_{a\nu}$$

$$+\xi^\nu\omega_{\nu a}{}^b e_{b\mu} + \Lambda_a{}^b e_{b\mu} + \delta_{\varepsilon'}e_{a\mu},$$

where $\delta_{\varepsilon'}$ a supersymmetric transformation with the parameter

$$\varepsilon' = -\tfrac{1}{2}K\xi\cdot\tau + \tfrac{1}{16}K(\xi^\mu\gamma_\mu + 2\xi^{ab}\sigma_{ab})(A + i\gamma_5 B)\chi,$$

and

$$\xi^\mu = 2\bar{\varepsilon}_1\gamma^\mu\varepsilon_2,\quad \xi^{ab} = 2\bar{\varepsilon}_1\sigma^{ab}\varepsilon_2,$$

$$\Lambda_{ab} = \tfrac{1}{4}K^2\varepsilon^{abcd}\xi_c(\bar{\chi}\gamma_5\gamma_d\chi)+\tfrac{1}{2}K(\xi_{ab}V_1-\tfrac{1}{4}\varepsilon_{abcd}\xi^{cd}V_2),$$

$V_1$ and $V_2$ are defined as $\hat{V} = V_1 + i\gamma_5 V_2$.

Let us consider some simple cases:

1./ First let us choose $\alpha$ to be linear: $\alpha=\lambda(\mu+Z)$, where $\lambda$ and $\mu$ are real. Then

$$\hat{V} = \lambda\,(+Z)e^{u/4},$$
$$\hat{A} = \tfrac{1}{2}\lambda(1 + \tfrac{u}{2} + \tfrac{1}{2}\mu\bar{Z})e^{u/4}, \qquad /5/$$
$$\hat{G} = \lambda K[(1 + \tfrac{u}{4})Z + \tfrac{1}{4}\mu\bar{Z}^2]e^{u/4},$$
$$U = \frac{\lambda^2}{8}\left[1 - \tfrac{3}{2}\mu^2 - 2\mu KA(1-\tfrac{u}{4}) + (\tfrac{\mu^2}{4} - \tfrac{1}{2})u + \tfrac{u^2}{4}\right]e^{u/2}$$

$\mu$ can be chosen in such a way that the absolute minimum of $U$ be zero. It can be seen, that at this minimum $B_o=0$ and $CK^{-1}=A_o$ where $0<\mu$, $c<2$. Introducing $A'=A-CK^{-1}$ we have the following terms in $L_i$:

$$-\tfrac{1}{2}\, em_\chi\bar{\chi}\chi - \tfrac{1}{2}em_A^2\, A'^2 - \tfrac{1}{2}em_B^2\, B^2 - \tfrac{1}{2}em_\tau\bar{\tau}_\mu\sigma^{\mu\nu}\tau_\nu \qquad /6/$$

$$+ \tfrac{1}{2}n\;\bar{\tau}\cdot\gamma\chi,$$

with

$$m_\chi = 2\lambda KC(1 + \tfrac{1}{4}\mu c + \tfrac{1}{4}\,c^2)e^{\frac{c^2}{4}},$$

$$m_{A'}^2 = \frac{1}{32}\lambda^2 K^2 (1 - \frac{1}{2}\mu^2 - \mu c - \frac{1}{2}c^2) e^{\frac{c^2}{2}} ,$$

$$m_B^2 = \frac{1}{32}\lambda^2 K^2 (1 - \frac{1}{2}\mu^2 - \mu c - c^2) e^{\frac{c^2}{2}} ,$$

$$m'_\tau = -2\lambda K(\mu + c) e^{\frac{c^2}{4}} , \qquad /7/$$

$$n = \frac{1}{2}\lambda K(1 + \frac{1}{2}\mu c + \frac{1}{2}c^2) e^{\frac{c^2}{4}} .$$

It is worthwhile to note that Higgs mechanism can be observed.

2./ Another interesting family of the functions $\alpha$ has the form: $\alpha = (Z - 1)^2 f(Z)$. In this case $\hat{V}(1) = \hat{A}(1) = 0$ and $U$ has a local minimum at $KA = 1$, $B = 0$. Thus for the shifted fields, in $L_i$ we fund the following terms:

$$-\frac{1}{2} em^2 A^2 - \frac{1}{2} em^2 B^2 - \frac{1}{2} em \bar{\chi}\chi ,$$

$$/8/$$

with $m = K f(1) \exp(\frac{1}{4})$.

The equality of the masses signifies that no spontaneous supersymmetry breaking happens.

If $f$ starts with a high enough power of $Z$, before the spontaneous supersymmetry breaking the mass spectrum is degenerate at zero. This is necessary to the reduction of the effect of the rather complicated operator equations (4) to the same simpler one of the algebre of [1]. In fact, if $f = Z^3$, the mass spectrum is degenerate of zero, moreover the state A=B=O is unstable.

In both considered cases the value of the cosmologic terms after the spontaneous symmetry breaking becomes equal to zero, thus the flat space-time beckground in the quantization procedure can be preserved.

Let us remark that starting from $\mathcal{L}=\mathcal{L}_o+\mathcal{L}_m$, one can achieve $\mathcal{L}=\mathcal{L}_o+\mathcal{L}_m+\mathcal{L}_i$ in the following two steps:

1./ Replacing in $\mathcal{L}_o+\mathcal{L}_m$ $D_\mu\tau_\nu$ and $D_\mu\chi$ with $D'_\mu\tau_\nu = D_\mu\tau_\nu + \frac{K}{8}\gamma_\mu\hat{V}\tau_\nu$ and $D'_\mu\chi = D_\mu\chi + \frac{K}{4}\gamma_\mu\hat{G}\chi + K\hat{A}\,\tau_\mu$, respectively. The term $K\hat{A}\tau_\mu$ is the correction according to the supercovariant derivative [3], and the terms

$\frac{K}{8}\gamma_\mu \hat{V}\tau_2$, $\frac{K}{4}\gamma_\mu \hat{G}\chi$ are similar to the ones of the de Sitter covariant derivative.

2./ Adding a spontaneous supersymmetry breaking potential $U(A,B)$.

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[6] After this work has been completed, in a preprint of A. Das, M. Fischler, and M. Rocek, ITP-SB-77-38 a more general Lagrangian was obtained.

[7] Conventions are $g_{\mu\nu} = (++++)$, $\gamma_a^2 = \gamma_5^2 = 1$,

$\gamma_a = \gamma_a^+$, $a = 1,4$, $\sigma_{\mu\nu} = \frac{1}{4}[\gamma_\mu, \gamma_\nu]$